Nuclear Engineering and Design 240 (2010) 132–138

Contents lists available at ScienceDirect

Nuclear Engineering and Design

journal homepage: www.elsevier.com/locate/nucengdes

Solution of the multiplying binary stochastic media based on L–P equation in 1D spherical geometry

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ARTICLE INFO

Article history:
Received 25 June 2009
Received in revised form 17 October 2009
Accepted 30 October 2009

ABSTRACT

In order to obtain the ensemble average k_{eff} for the binary stochastic media system, a statistical transport equation with eigenvalue is derived based on the L–P equation. Combined method of statistical and deterministic is proposed to deal with this problem. In the first step, Monte Carlo approach is employed to calculate the mean chord length of the background scattering material, deterministic transport method using diamond difference and source iteration is applied to solve the eigenvalue L–P equation. Test problems of different scattering ratios, different chord-path ratios and different number of random fissile lumps and simple UO_2–H_2O problem are calculated and compared with references. Results show that the eigenvalue L–P equation can provide the mean value of k_{eff} for the given stochastic systems in most cases.

1. Introduction

Particle and radiation transport in the stochastic medium has been widely applied in radiative transfer, atmospheric sciences and considerable interests in nuclear criticality safety, nuclear waste disposal and repository, numbers of methods have been advised and developed. Statistical analyses are carried out based on transport or diffusion theory accounting for appropriate statistical approximation. The statistics of flux and multiplication factor are obtained for the random system through either analytical or numerical means (Williams, 2000, 2002, 2003). L–P model (Adams et al., 1989; Pomraning, 1989; Malvagi and Pomraning, 1992; Su and Pomraning, 1993; Sanchez, 1989, 2006, 2008a,b; Akcasu and Williams, 2004; Akcasu, 2007) is developed based on an exact stochastic equation for material fluxes and on a approximate relation. The exact equation contains interface fluxes which includes the ensemble averages over realizations that the fluxes change from one material to the other at a given spatial location. The approximate relation replaces the complicated interface fluxes by the material fluxes from the upstream material. This approximation is exact in pure-absorbing material with Markov chord length statistics and this is extended to the cases with scattering. Alternating renewal theory is chosen for non-Markov statistics. Davis and Palmer (2005) have given out the benchmark for two-group k-eigenvalue calculation in planar geometry. In their work, the problem is comprised of fuel and moderator. The size of the problem and the mean chord length of the fuel or moderator are determined by chord length sampling of a PWR assembly. Markov chord length distribution and sampling is employed to determine the random thickness of alternating fuel and moderator in the 1D slab system. Different approaches of chord length sampling are intended (e.g. for moderator, Markov or Matrix; for fuel, disk or Markov). The ensemble average flux and the PDF of the system k_{eff} is eventually obtained by performing sufficient numbers of the calculation. Nevertheless, no eigenvalue problem for multiplying media was proposed concerning straightforward solution of the LP model to estimate the ensemble average k-eigenvalue.

In this paper, an eigenvalue statistical transport equation is proposed by adding particular fission term and eigenvalue to the L–P equation. Spherical outer boundary scattering background system consists of number of randomly distributed fissile pellets is studied. Mean chord length of the background material is obtained by Monte Carlo approach under homogeneous Markov chord length distribution approximation and the ensemble average k_{eff} of the system is calculated by straightforward solution of the eigenvalue L–P equation in 1D spherical geometry.

In Section 2 the model of eigenvalue L–P equation is explained. Section 3 illustrates the generation of the random system, Section 4 provides the solution of the mean chord length. In Section 5, numerical results are given and compared with the reference. Conclusion and remarks are made in the final section.

2. Eigenvalue L–P equation

Fig. 1 shows a binary stochastic system with fissile material i randomly distributed in the non-fissile scattering background material

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0029-5493/$ – see front matter
doi:10.1016/j.nucengdes.2009.10.027

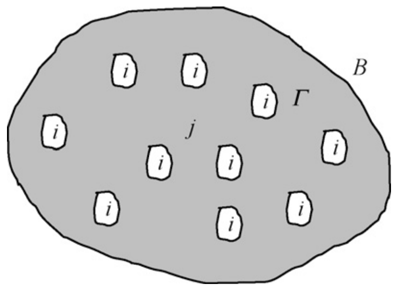

Fig. 1. Binary stochastic media system with random distributed fissile lumps.

j. B is the outer boundary of the convex volume V and Γ is the interior surface that separates two materials.

2.1. The L–P equation

$$\begin{aligned}&\Omega\cdot\nabla(\psi_i(r,\Omega))+\Sigma_{ti}\psi_i(r,\Omega)+\frac{1}{\lambda_i}(\psi_i(r,\Omega)-\psi_j(r,\Omega))\\&=\frac{1}{4\pi}\Sigma_{si}\int_{4\pi}\psi_i'(r,\Omega')d\Omega'+Q_i(r,\Omega)\end{aligned}\tag{1}$$

Eq. (1) is the steady-state, one group, isotopic scattering L–P equation (Adams et al., 1989) for material i in binary stochastic media. It is given under the assumption of Markov mixing statistics for the two components in the system. Further, the statistics are taken as homogeneous, by which it means that all points in the system have the same statistical properties. In the L–P equation, material fluxes ($\psi_i(r,\Omega)$, $\psi_j(r,\Omega)$) are used to replace the interface fluxes ($\psi_{ij}(r,\Omega)$, $\psi_{ji}(r,\Omega)$) in the coupling term (true for pure-absorbing mixture and extended to cases with collision). The ensemble average flux is given as

$$\left\langle\psi(r,\Omega)\right\rangle=\psi_i(r,\Omega)p_i+\psi_j(r,\Omega)p_j\tag{2}$$

where for homogeneous statistics, we have

$$p_i=\frac{\lambda_i}{\lambda_i+\lambda_j},\quad p_j=\frac{\lambda_j}{\lambda_i+\lambda_j}\tag{3}$$

$\psi_i(r,\Omega)$ is the material angular flux for material i. p_i is the emerging probability that finding material i at a spatial point r in the system. The notation $\langle\cdot\rangle$ indicates ensemble averaging operation, λ_i is the mean chord length for material i and Σ_{it}, Σ_{si}, are total cross-section, scattering cross-section, for material i, respectively. $\langle\psi(r,\Omega)\rangle$ is the ensemble average angular flux.

The boundary condition of Eq. (1) is non-stochastic vacuum or reflective, written as

$$\psi_i(R,\Omega)=\begin{cases}0,\ \Omega\cdot n<0, & \text{for vacuum boudary}\\ \psi_i(R,\Omega), & \text{for reflective boudary}\end{cases}\tag{4}$$

2.2. Eigenvalue of the L–P equation

Fission term and eigenvalue are introduced to Eq. (1). We have

$$\begin{aligned}&\Omega\cdot\nabla(\psi_i(r,\Omega))+\Sigma_{ti}\psi_i(r,\Omega)+\frac{1}{\lambda_i}(\psi_i(r,\Omega)-\psi_j(r,\Omega))\\&=\frac{1}{4\pi}\Sigma_{si}\int_{4\pi}\psi_i'(r,\Omega')d\Omega'+\frac{\nu\Sigma_{fi}}{k}\int_{4\pi}\psi_i'(r,\Omega')d\Omega'\end{aligned}\tag{5}$$

where k is the eigenvalue of the equation, $\nu\Sigma_{fi}$ is the fission production cross-section of material i, isotropic fission is assumed. The only differences between Eq. (5) and Eq. (1) are the fission term and eigenvalue k. The detail procedure deriving Eq. (5) is not discussed here because it is similar to the derivation of Eq. (1) in sense of neutron balance.

To explain the fission term and eigenvalue, ensemble averaging is defined as follows:

$$\psi_i(r,\Omega)=\frac{\int_{K_i(r)}p(\omega)\psi_\omega(r,\Omega)d\omega}{\int_{K_i(r)}p(\omega)d\omega}=\frac{\int_{K_i(r)}p(\omega)\psi_\omega(r,\Omega)d\omega}{p_i(r)}\tag{6}$$

where $\psi_i(r,\Omega)$ is the ensemble average material flux at position r. The averaging is carried out on the subset $K_i(r)=\{\omega\in K,\ \omega(r)=i\}$ of realizations that have material i at position r and $\psi_\omega(r)$ is the flux for random realization ω. K represents the statistical set of all physical realizations. $p(\omega)$ denotes the density of probability for realization ω.

Performing ensemble averaging on the fission source for material i in the stochastic system, we have

$$F(r,\Omega)=\frac{\int_{K_i(r)}p(\omega)\frac{Q_w(\psi_\omega(r))}{k_\omega}d\omega}{p_i(r)}=\frac{Q_i\left(\int_{K_i(r)}p(\omega)\frac{1}{k_\omega}\psi_\omega(r)d\omega\right)}{p_i(r)}\tag{7}$$

where $Q_\omega(\bullet)$ is the fission source calculation operator and k_ω is the k_{eff} of the system for an individual realization ω. As the fission cross-section of material j is 0, fission term exists only when spatial point r is in material i, $Q_\omega(\bullet)$ is therefore replaced by $Q_i(\bullet)$. To make the material fission source and eigenvalue easy to handle, we assume that

$$\int_{K_i(r)}p(\omega)\frac{1}{k_\omega}\psi_\omega(r)d\omega\approx\frac{1}{k}\int_{K_i(r)}p(\omega)\psi_\omega(r)d\omega\tag{8}$$

Eq. (8) is finally written as

$$F(r,\Omega)=\frac{Q_i(\psi_i(r,\Omega))}{k}=\frac{1}{k}\frac{\nu\Sigma_{fi}\int_{4\pi}\psi_i'(r,\Omega')d\Omega'}{4\pi}\tag{9}$$

Eq. (9) gives the fission term involving eigenvalue k that is invariant of realizations agreed with Eq. (5), representing ensemble average k_{eff} of the system.

2.3. Equivalence of the eigenvalue k and ensemble average k_{eff}

According to the definition of the ensemble averaging operation in Eq. (6), we can give the exact expression of the ensemble average k_{eff} of the system, viz.

$$k_{en}=\frac{\int_{K_i(r)}p(\omega)k_\omega d\omega}{\int_{K_i(r)}p(\omega)d\omega}\tag{10}$$

Inserting Eq. (8) into Eq. (7), eigenvalue k is expressed as

$$k=\frac{\int_{K_i(r)}p(\omega)Q_w(\psi_\omega(r))d\omega}{\int_{K_i(r)}p(\omega)\frac{Q_w(\psi_\omega(r))}{k_\omega}d\omega}\tag{11}$$

If we ensure that $Q_w(\psi_\omega(r))/k_\omega=1$, which can be satisfied by the normalization of the fission source term in the numerical solution of eigenvalue equation. Eq. (11) is then identical to Eq. (10) so that k is proven equivalent to the ensemble average k_{eff} of the system. Eq. (5) is readily to be solved to obtain the ensemble average k_{eff} of the system represented by eigenvalue k.

3. Construction of the stochastic media system

In the present paper, spherical outer boundary system is studied (Fig. 2). The system contains two components, the first is numbers of randomly dispersed fissile pellets and the second is the scattering background. The main steps of creating the stochastic system are:

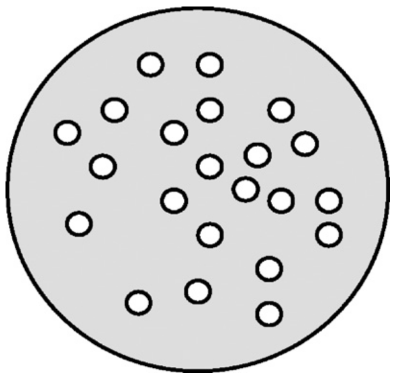

Fig. 2. Randomly distributed fissile pellets in the spherical region.

1. Locations of the fissile pellets in the system are determined sequentially by sampling of uniform distribution random number.
2. Randomly placed position of the pellets that overlap one another or the surface of the system boundary are rejected and regenerated.
3. Steps 1 and 2 are repeated until required numbers of the fissile pellets are randomly placed in the given system region.

Uniform distribution random number sampling guarantees the material mixing statistics to be homogeneous. Thus, the material emerging probability is universal in the system if the edge effect is neglected.

The distribution of the pellets' positions may vary greatly case by case along the radius from one direction to another. However, after the averaging over all physical realizations, the homogeneous mixing statistics ensures the material emerging probability and the ensemble average material flux behave consistently in arbitrary radial direction from the center of the spherical system. Therefore, material flux and other statistical sets are independent of centric rotation of the system because rotation can be regarded as a particular one among all realizations, which does not affect the overall statistics. We hence rewrite Eq. (5) in 1D symmetric spherical coordinate

$$\frac{\mu}{r^2}\frac{\partial \psi_i(r,\mu)}{\partial r}+\frac{1}{r}\frac{\partial(1-\mu^2)\psi_i(r,\mu)}{\partial \mu}+\Sigma_{ti}\psi_i(r,\mu)+\frac{1}{\lambda_i}[\psi_i(r,\mu)-\psi_j(r,\mu)] = \frac{1}{2}\left(\Sigma_{si}+\frac{\nu\Sigma_{fi}}{k}\right)\int_{-1}^{1}\psi_i'(r,\mu')d\mu' \quad (12)$$

3D randomized binary system is now characterized by 1D spherical symmetric geometry based on the ensemble averaging and homogenous statistics.

4. Solution of the mean chord length

The mean chord length for both randomly positioned pellets and scattering background must be calculated beforehand in order to proceed the deterministic numerical solution of Eq. (12). In our study, all fissile pellets are in the same size. The mean chord length of one single pellet is derived analytically while the mean chord length of the scattering background is calculated by Monte Carlo approach due to its complexity.

4.1. Chord length distribution of one single pellet

Let us assume a ray representing the particle travel path through one fissile pellet (see Fig. 3) whose radius is r, Su and Pomraning (1993) has suggested the chord length distribution in the pellet follows the probability density function

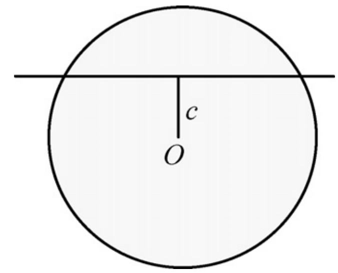

Fig. 3. Random chord through the stochastic fissile pellet.

$$f(l)=\frac{dF(l)}{dl}=\frac{l}{2r^2} \quad (13)$$

the mean chord length of the pellet is

$$\lambda_i=\int_0^{2r} lf(l)dl=\frac{4r}{3} \quad (14)$$

All the random spheres have the same mean chord length because of their same radius and the homogenous statistics of the system.

4.2. Chord length distribution of the background

In modeling the particle transport, it is often assumed that the distribution of chord length (distance between the random fissile pellets) is Markov. In other words, along any ray through a medium, the material transitions are described by Poisson statistics, so that the probability of a transition from material i to material j in a short distance d_s is d_s/λ_i. This corresponds to an exponential distribution of material chord lengths with the mean chord length λ_i. Olson et al. (2006) has demonstrated that non-overlapping spheres randomly positioned in 2D square or 3D cubic region, the distribution of chord lengths in the background region is accurately exponential when the material is dilute (less than 10% volume packing fraction). Torquato (2002) showed that the chord length (l) distribution between the pellets in the spherical background region also can be

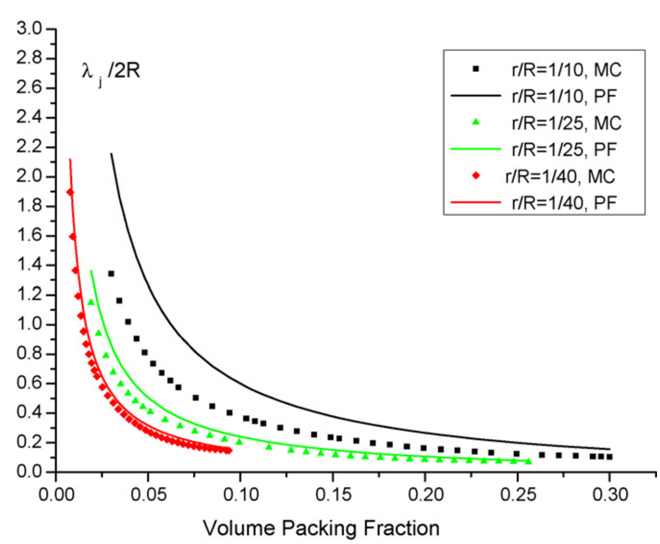

Fig. 4. Background mean chord length for different r/R versus volume packing fractions.

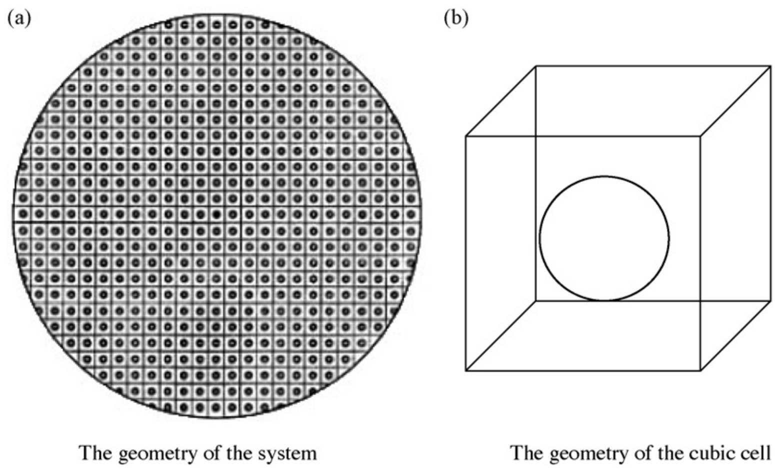

Fig. 5. Geometry of the lattice model (a) the geometry of the system and (b) the geometry of the cubic cell.

represented by the exponential form

$$p(l) = \frac{1}{\lambda_j} \exp\left(-\frac{l}{\lambda_j}\right) \tag{15}$$

where λ_j is the mean chord length of the background region.

A chord length probability density function (PDF) is obtained by Ji and Martin (2007) assuming the PDF is exponential which is true for Markov statistics

$$p(l) = \frac{3}{4r} \cdot \frac{PF}{1-PF} \exp\left(-\frac{3}{4r} \cdot \frac{PF}{1-PF} \cdot l\right) \tag{16}$$

where r is the radius of the random fissile pellet and PF is the volume packing fraction in the system.

By the PDF given in Eq. (16), the mean chord length of the background is

$$\lambda_j = \int_0^\infty lp(l)dl = \lambda_i \cdot \frac{1-PF}{PF} \tag{17}$$

The derivation is based on binary statistics on the line, in 3D geometry; the distribution is identical to Eq. (16). Excellent agreement was obtained for the chord length sampling (CLS) to predict the transport of resonance energy neutrons incident on a large box of TRISO micro spheres with various packing fractions characteristic of both pebble bed and prismatic VHTR designs. It is quite accurate for the large system containing relatively small size grains; however the edge effect is not negligible for systems while the size of the stochastic pellets are relatively large. Donovan and Danon (2003a) have proposed a statistical technique based on Monte Carlo method to get the mean chord length of the background material in 2D rectangle region with numbers of randomly placed disks.

In Donovan's method, random system is firstly built according to the method suggested in Section 3, N_l random rays are then drawn from the center of the system, n_i rays that has no random pellets overlapping are tallied. This building-drawing-tallying process is repeated for large number of N times sufficiently to characterize the statistical property, the probability P_0 that a ray in arbitrary direction starting from the center of background region would encounter no stochastic pellets along its path is calculated as

$$P_0 = \lim_{N\to\infty} \frac{\sum_{i=1}^{N_l}(n_i/N_l)}{N} \tag{18}$$

given that the PDF of background chord length follows Eq. (15), Donovan et al. (2003b) suggested a more sophisticated relation between P_0 and λ_j for spherical region

$$P_0 = \int_{R-r}^{\infty} \frac{1}{\lambda_j} e^{(-x/\lambda_j)} dx + \int_{R-2r}^{R-r} \frac{1}{\lambda_j} e^{(-x/\lambda_j)} \left(\int_{2(R-r-x)}^{2r} \frac{y}{2r^2} dy \right) dx \tag{19}$$

In this paper, the mean chord length of the background is obtained by solving Eq. (19) after the P_0 is calculated by Monte Carlo approach in Eq. (18).

5. Numerical results

5.1. Mean chord length of the background material

Mean chord length of the background region of the system in Fig. 2 is calculated, for different r/R ratios and different number of random pellets, where r is the radius of the fissile pellet, R is the radius of the background region. In this calculation, N is chosen to be 10^6.

Fig. 4 compares background mean chord length results based on Eq. (17) (marked as PF) and Eq. (19) (marked as MC) for different r/R and volume packing fractions. It is observed that the PF model overestimates the background mean chord length therefore the fissile material emerging probability in Eq. (3) is underestimated. The gap between two models becomes smaller as the r/R ratio decreases because system with smaller r/R ratio is less influenced by edge effect.

5.2. Ensemble average k_{eff}

In order to investigate the eigenvalue L–P equation accuracy of estimating the ensemble average k_{eff}, reference results are provided the by brutal-force simulation. The stochastic system is constructed by the approach proposed in Section 3. After each random system is created, multi-group MCNP4C (Briesmeister, 2000) is employed to do the transport calculation. The ensemble averaged k_{eff} of the system is obtained by large number of the random system rigorous calculation.

In this study, 10^5[i] random systems are intended to get the ensemble average k_{eff} (denoted $\langle k_{\text{eff}} \rangle_{\text{Ref.}}$). For each individual realization, 200 generations of 3000 particles are tracked for the Monte Carlo calculation.

[i] In practice, this involves unbearable time consumption for doing such number of realization. Hence we run as few realizations as we can if only the ensemble average k_{eff} converges. It took about 10^3 realizations to provide quite accurate average k_{eff}.

Table 1
Parameters for different scattering ratio calculations (5% volume packing fraction).

r (cm)	R (cm)	N	Σ_n (cm^{-1})	$\nu\Sigma_{fi}$ (cm^{-1})	Σ_{ti} (cm^{-1})
2	20	50	3.74	0.374	0.037

A stochastic media statistical transport (SMST) computation code is developed based on deterministic S_N method using diamond difference and source iteration to solve the eigenvalue L–P equation in 1D spherical geometry, the boundary condition is non-stochastic vacuum as in Eq. (4). The eigenvalue (denoted $\langle k_{\text{eff}}\rangle_{\text{SMST}}$) for the equation is calculated by power iteration.

In another calculation, more crude model is used: the randomness in the fissile sphere distribution is ignored. The positions of the random spheres are fixed in the background region. Lattice geometry is considered based on the total volume fraction of the system. The entire region is filled with cubic cell, which has the same volume fraction with the system (shown in Fig. 5). The result k_{eff} (denoted $\langle k_{\text{eff}}\rangle_{\text{Lat.}}$) is calculated by MNCP, 200 generations of 3000 particles are tracked.

One group isotropic scattering and fission cases are chosen for simplicity. Problems of different scattering ratios, different chord-path ratios and different number of the random spheres are calculated and discussed. In the end, a simplified realistic problem is calculated and compared to show the great difference between the eigenvalue L–P equation and lattice model.

5.2.1. Different scattering ratios

The scattering ratio of material i is changed from 0.9 to 0.0 and the scattering ratio of material j is changed from 1.0 to 0.1; the number (N) and the radius of random pellets (r) and the radius of the background sphere (R) as well as total cross-section are fixed (shown in Table 1).

Table 2 shows the results calculated by the eigenvalue L–P equation and the lattice model and their relative error. It is seen that the eigenvalue L–P equation provides slightly better prediction for the ensemble average k_{eff} than the lattice model. Smaller scattering ratio of the fissile and background material cases give more accurate results to the reference.

5.2.2. Different chord-path ratios

Chord-path ratio (CPR) is defined as the ratio of mean chord length to mean free path of a particular material. In this section, r, R, N and scattering ratio of material i and j are fixed for the cases (shown in Table 3), the total cross-sections of material i and j are changed accordingly to satisfy the desired chord-path ratio.

The mean chord length represents the chances that neutron encounter the material while the mean free path stands for the probability that neutron may interact within the material. Thus, the chord-path ratio reveals the relations between these two probabilities.

Results in Table 4 show that the eigenvalue L–P model accuracy is greatly influenced by the chord-path ratio of the materials. Errors are augmented as the chord-path ratio of the fissile pellets decreases although the accuracy is quite good when the chord-path ratio is large. Changing the chord-path ratio of one material does not affect the accuracy when the chord-path ratio of the other material is large. But if the chord-path ratios of both materials are small, the results become unacceptably worse.

Table 3
Parameters for different chord-path ratio calculations (6.4% volume packing fraction).

r (cm)	R (cm)	N	Σ_{si}/Σ_{ti}	$\nu\Sigma_{fi}/\Sigma_{ti}$	Σ_{sj}/Σ_{tj}
2	50	1000	0.5	0.1	1.0

Table 4
Ensemble average k_{eff} of different chord-path ratios.

Case	CPR$_i$	CPR$_j$	$\langle k_{\text{eff}}\rangle_{\text{Ref.}}$	$\langle k_{\text{eff}}\rangle_{\text{SMST}}$	$\langle k_{\text{eff}}\rangle_{\text{Lat.}}$	Err. (%) SMST/Lat.
5	10	10	0.1888	0.1891	0.1892	0.16/0.21
6	10	1.0	0.1842	0.1844	0.1839	0.11/−0.16
7	5.0	10	0.1847	0.1855	0.1841	0.43/−0.32
8	5.0	1.0	0.1724	0.1767	0.1710	2.49/−0.81
9	1.0	10	0.1534	0.1591	0.1499	3.71/−2.28
10	1.0	1.0	0.1063	0.1293	0.1015	21.6/−4.52

Table 5
Material i and j for cases with different number of random spheres.

$\nu\Sigma_{fi}/\Sigma_{ti}$ (cm^{-1})	Σ_{si}/Σ_{ti}	Σ_{ti} (cm^{-1})	Σ_{sj}/Σ_{tj}	Σ_{tj} (cm^{-1})
0.1	0.5	3.74	1.0	0.3

Table 6
Ensemble average k_{eff} of different number random spheres (5% volume fraction).

Case	r, R, N	$\langle k_{\text{eff}}\rangle_{\text{Ref.}}$	$\langle k_{\text{eff}}\rangle_{\text{SMST}}$	$\langle k_{\text{eff}}\rangle_{\text{Lat.}}$	Err. (%) SMST/Lat.
11	2, 20, 50	0.1823	0.1825	0.1811	0.16/−0.66
12	1.587, 20, 100	0.1792	0.1803	0.1776	0.61/−0.89
13	1, 20, 400	0.1738	0.1756	0.1719	1.04/−1.09
14	0.5, 20, 3200	0.1710	0.1729	0.1674	1.11/−2.11

5.2.3. Different numbers of the random fissile spheres

Systems comprised of different numbers of the fissile spheres are constructed and calculated. Fissile and the background materials are listed Table 5 and total volume fraction is fixed (5%). The number (N) and the size (r) of the random fissile pellets are changed correspondingly to maintain the fixed total volume fraction.

Great numbers of smaller random fissile spheres compose system with relatively high homogeneity. It gives lower ensemble average k_{eff} than the system with less number of lager size fissile spheres. In addition, a homogeneous mixture according to the volume fraction is calculated, the k_{eff} is 0.1674, this mixture can be identified as the system containing ultimate large amount of vanishingly small random fissile grains.

Table 6 shows that the ensemble average k_{eff} decreases as the size random fissile pellets diminish. The relative errors of both models become larger as the size shrinks. The eigenvalue L–P model is more accurate than the lattice model in general and overestimation and underestimation of the ensemble average k_{eff} are observed constantly in these two models, respectively.

Table 2
Ensemble average k_{eff} of different scattering ratios.

Case	Σ_{si}/Σ_{ti}	Σ_{sj}/Σ_{tj}	$\langle k_{\text{eff}}\rangle_{\text{Ref.}}$	$\langle k_{\text{eff}}\rangle_{\text{SMST}}$	$\langle k_{\text{eff}}\rangle_{\text{Lat.}}$	Err. (%) SMST/Lat.
1	0.9	1.0	0.7146	0.7275	0.6991	1.81/−2.17
2	0.9	0.1	0.6974	0.7038	0.6885	0.92/−1.28
3	0.0	1.0	0.0963	0.0961	0.0960	−0.21/−0.31
4	0.0	0.1	0.0960	0.0959	0.0958	−0.10/−0.21

Table 7
Parameters for simplified test problem (9.138% volume packing fraction).

r (cm)	R (cm)	N	Fissile pellets	Background material	Boundary condition
1.568	15	80	UO_2 (^{235}U 15%W/O)	H_2O	Vacuum

Table 8
Ensemble average k_{eff} of the simplified test case.

$\langle k_{eff}\rangle_{Ref.}$	$\langle k_{eff}\rangle_{SMST}$	$\langle k_{eff}\rangle_{Lat.}$	Err. (%) SMST/Lat.
0.9210	0.9250	0.8498	0.43/−7.73

5.3. Simplified test problem

In this section, a simplified sub-critical problem is presented. The system in Fig. 2 consists of randomly distributed UO_2 pellets in the water moderating background. Problem size and materials are given in Table 7. WIMS 69 multi-group library is used in the calculation and the self-shielding effect is taken into account.

Table 8 gives out the results calculated for the simplified problem. Great difference is seen in the lattice model while the eigenvaule L–P equation agrees well with the reference. In the calculation, fissile pellets distribution uncertainty is not considered for the self-shielding effect; same effective resonance cross-sections are prepared for both eigenvalue L–P and lattice calculation.

Fig. 6 shows the emerging probability of fissile material i along radial direction of the system. The actual distribution (black curve) is obtained by Monte Carlo approach. Compared with the actual probability, homogenous statistics in Eq. (3) gives slightly higher estimate in the red curve (For interpretation of the references to color in this text, the reader is referred to the web version of the article.) in most region, which leads to higher ensemble average k_{eff}. The lattice model fixes all the positions of the pellets; ignoring those places that the fissile pellets may appear where they can contribute to higher k_{eff}. In addition, the average emerging probability of the fissile material is plotted as the green curve. It is obtained by integrating the actual probability in the entire region and averaged by the total volume. It equals to the total volume fraction of the fissile material. It is clearly shown that this average probability is much lower in most region than the actual value; if we use this average probability to quantify the mean chord lengths of system, it will be the same as in Eq. (17), eigenvalue L–P model using this mean chord lengths gives the result as 0.7769.

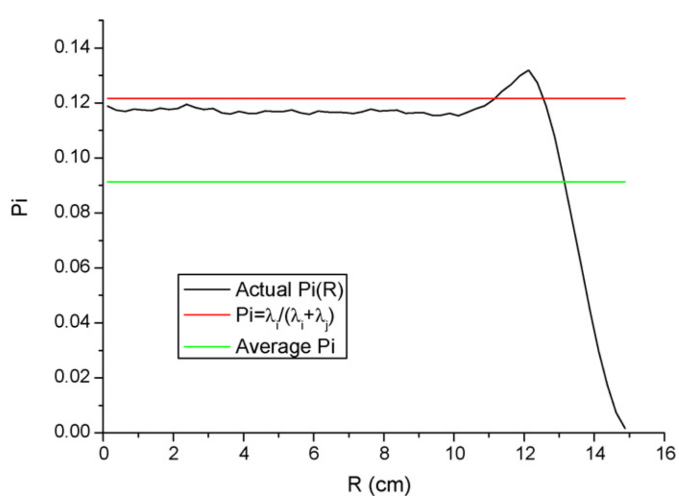

Fig. 6. The emerging probability of fissile material along radial direction.

6. Conclusions and remarks

In this study, an eigenvalue is introduced to the L–P equation. By assuming the background material chord length distribution obey homogeneous Markov statistics, mean chord length of background material is obtained by Monte Carlo approach. Numerical solution of Eq. (12) is carried out using simplified 1D symmetric spherical geometry. Results show that the eigenvalue L–P equation is able to predict the ensemble average value of the k_{eff} in most given cases.

It can be concluded from the results that the scattering ratio of both material do not greatly influence the precision of the eigenvalue L–P equation. The accuracy of the results is more sensitive to the chord-path ratio. The relative errors of ensemble average k_{eff} become absurd if the chord-path ratios of both materials are small.

It is also found that if materials and total volume fraction is fixed, eigenvalue L–P equation is more reliable for the systems with lower density of larger size fissile pellets than the system containing great numbers of smaller pellets. Eigenvalue L–P model overestimates the ensemble average k_{eff} in all given cases due to the overestimation of the fissile material emerging probability by the homogenous statistics.

In general, simple geometry and statistics are chosen to illustrate the straightforward solution of the ensemble average k_{eff} by eigenvalue L–P equation. Great deficiency is observed when the chord-path ratios of both materials are small. It is necessary to develop more sophisticated statistics to improve inappropriate homogeneous Markov statistics for spherical geometry.

Acknowledgement

This work is supported by the National Natural Science Foundation of China Grants Nos. 10475064 and 10605017.

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